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Localization properties of two interacting electrons in a disordered quasi one-dimensional potential

Jean Richert[1] **and Hans A Weidenmüller**[2]

[1] Laboratoire de Physique Théorique[3], Université Louis Pasteur, 67084 Strasbourg Cedex, France
[2] Max-Planck-Institut für Kernphysik, D-69029 Heidelberg, Germany

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Abstract

We study the transport properties of two electrons in a quasi one-dimensional disordered wire. The electrons are subject to both a disorder potential and a short range two-body interaction. Using the approach developed by Iida *et al* (1990 *Ann. Phys., NY* **200** 219), the supersymmetry technique and a suitable truncation of Hilbert space, we work out the two-point correlation function in the framework of a nonlinear σ model. We study the loop corrections to arbitrary order. We obtain a remarkably simple and physically transparent expression for the change of the localization length caused by the two-body interaction.

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Non-interacting electrons in a one-dimensional random potential are localized. The presence of a short-range two-body interaction (a screened Coulomb potential) affects the localization length. For two interacting electrons in one dimension, this surprising fact, originally discovered by Shepelyansky [1], has been clearly established in a series of numerical and analytical investigations (see [2–5] and references therein).

It is obviously most desirable to extend these investigations to two-dimensional systems. At present, numerical approaches do not seem capable of handling the ensuing difficulties. However, the work in [1, 2, 4] yields an analytical expression for the change of the localization length $\zeta(U)$ under the influence of a two-body interaction with typical matrix element U. The result

$$\frac{\zeta(U)}{\zeta(0)} - 1 \propto \left(\frac{U}{B}\right)^2 \tag{1}$$

with B the bandwidth due to the disorder potential, is independent of the sign of the two-body interaction and is claimed to hold both in one and in two dimensions. Numerical results in one dimension [3], although in line with the sign-independence, do not fully support the U^2 dependence predicted analytically. Moreover, the effect is numerically found to be strongest

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when the two electrons move at short distance [5], a feature which is also not covered by the analytical expression. Finally, the result (1) has either been derived heuristically [2], or it is based on statistical assumptions [4] which, although plausible, are not obviously valid.

This situation calls for a novel approach to the problem. In the present work we do not consider the one-dimensional case. Rather, we address the more realistic and more demanding case of two electrons in a quasi one-dimensional disordered wire interacting via a short-range interaction. Using an analytic method developed recently [6], a reduction of Hilbert space and supersymmetry, we achieve a complete analytical solution of the problem. We show that the two-body interaction affects the localization length via the level density. We establish a criterion for the onset of interaction-induced effects on the localization length. In the spirit of relation (1) we derive a formula for the change of the localization length with the two-body interaction. We discuss qualitatively the resulting change. Our final analytical expression allows for the numerical determination of the localization length for any given two-body interaction. To the best of our knowledge, this is the first time that a full analytical solution of a problem involving both disorder and interaction has been achieved.

We felt that the special issue devoted to random-matrix theory would be a suitable place to publish our results. However, space limitations make it impossible for us to give a full account of our work. Thus, we attempt to render a description which—we hope—is accessible to a broad audience. Technical details will be found in a forthcoming paper [7].

1. Model

The model we use was first formulated for non-interacting electrons in [8]. It is a discrete model in which electrons undergo a hopping motion. The model was shown to lead to the same supersymmetric nonlinear sigma model as obtained from a continuum model using a kinetic term and a random impurity potential V_{imp}. Hence, the model is generic. The introduction of a two-body interaction into the model is not expected to alter this conclusion.

We consider a quasi one-dimensional disordered wire of length L. The wire is thought of [8] as consisting of K slices labelled $a, b, c, \ldots, K$. The surfaces separating neighbouring slices are transverse to the axis of the wire and to the direction of the current. All slices are of equal size. Eventually we take the limit where $K \to \infty$ and where the longitudinal extension of the slices goes to zero. Disorder is modelled by assuming that the Hamiltonian within each slice is a member of a suitable ensemble of random matrices. The kinetic energy of the electrons is modelled in terms of hopping matrix elements connecting neighbouring slices. Two electrons move through the wire. A short-range two-body interaction acts whenever the two electrons occupy the same slice. To make the model manageable, we need to reduce the Hilbert space. We do so by admitting only states where the two electrons are either in the same slice, or occupy neighbouring slices.

Without this last step, our model would be very general. It is, therefore, important to address the physical significance of the last step. The numerical work in [5] in one dimension has clearly demonstrated that the change of the localization length caused by the two-body interaction is biggest when the two electrons which move through the one-dimensional wire, keep the shortest distance from each other. This is our motivation for the constraint artificially imposed in our model. It might have been desirable to loosen the constraint and to allow the electrons to keep the maximum distance given by the localization length in the absence of disorder. However, this was technically impossible. Therefore, we cannot claim that

our results are quantitatively correct. However, in view of the work in [5], we believe that we obtain qualitatively correct answers. The shortcoming of our approach—the reduction of Hilbert space—must be weighed against the fact that, with this simplification, we are able to obtain a complete analytical solution and a physically transparent answer to the problem.

We turn to the technicalities. In each slice we use an arbitrary basis of single-particle states labelled $|aj\rangle$ with $j = 1, 2, \ldots, l$ and fermionic creation and annihilation operators $\alpha^{\dagger}_{aj}$ and α_{aj}, respectively. We later take the limit $l \to \infty$. The Hamiltonian H is the sum of three terms, $H = H_0 + H_1 + H_2$. In every slice, disorder is simulated by an ensemble of random single-particle Hamiltonians belonging to the unitary ensemble,

$$H_0 = \sum_{aij} h^{(a)}_{ij} \alpha^{\dagger}_{ai} \alpha_{aj}. \tag{2}$$

The complex random variables $h^{(a)}_{ij}$ have a Gaussian distribution with mean value zero and the following non-vanishing second moments,

$$\overline{h^{(a)}_{ij} h^{(a')*}_{i'j'}} = \frac{\lambda^2}{l} \delta_{aa'} \delta_{ii'} \delta_{jj'}. \tag{3}$$

The overbar denotes the ensemble average and λ has the dimension of energy. The disorder Hamiltonians in different slices are uncorrelated. We use the unitary (rather than the orthogonal ensemble) to simplify the algebra. Hopping between neighbouring slices is described by the term

$$H_1 = \sum_{ai} v\big[\alpha^{\dagger}_{ai} \alpha_{a+1i} + \text{h.c.}\big]. \tag{4}$$

The short-range two-body interaction acts only between electrons in the same slice and reads

$$H_2 = \sum_{a} \sum_{i<j, i'<j'} w_{iji'j'} \alpha^{\dagger}_{ai} \alpha^{\dagger}_{aj} \alpha_{aj'} \alpha_{ai'}. \tag{5}$$

The two-body matrix elements are antisymmetric in the pairs (ij) and $(i'j')$ and Hermitean. Both v and the $w_{iji'j'}$ are non-random quantities independent of the slice label a.

The Hilbert space of the two-electron problem is spanned by the orthonormal states $|ab\mu\rangle$ where $a \leqslant b$ and where μ stands for the pair (i, j) with $i < j$ for $a = b$. For $a < b$ fixed, the number N_{ab} of states is l^2 while for $a = b$, $N_{aa} = l(l-1)/2$. The totality of states $|ab\mu\rangle$ with $\mu = 1, \ldots, N_{ab}$ is referred to as the box (ab). As discussed above, we reduce the Hilbert space by considering only the $2K - 1$ boxes where the two electrons are either located in the same slice a (this is box (aa) with $a = 1, \ldots, K$), or in the two adjacent slices a and $a + 1$ (this is box $(a(a+1))$, with $a = 1, \ldots, (K-1)$).

Localization properties can be read off the two-point correlation function

$$C(n) = \overline{|\langle a(a+1)\mu|(E^{+} - H)^{-1}|(a+n)(a+n+1)\nu\rangle|^2}. \tag{6}$$

To be independent of edge effects, we choose $1 \ll a \ll (a+n) \ll K$. For large n, $C(n)$ should decay exponentially in n. In proper units, the coefficient in the exponent defines the localization length. We calculate $C(n)$ using supersymmetry [9, 10].

1.1. Second moment of H_0

The average in equation (6) is to be taken over the distribution of the coefficients $h^{(a)}_{ij}$ in equation (2). This distribution is Gaussian and so is, therefore, the distribution of the matrix elements $\langle ab\mu|H_0|ab\sigma\rangle$ of H_0 with $b = a, (a+1)$. The ensemble average in equation (6)

is, thus, entirely determined by the second moments of these matrix elements, i.e. by the quantities

$$\mathcal{A}^{(k)}_{\mu\nu;\rho\sigma} = \frac{l}{\lambda^2}\overline{\langle ab\mu|H_0|ab\sigma\rangle\langle a'b'\rho|H_0|a'b'\nu\rangle}. \tag{7}$$

The index k labels the various possible combinations of (a, b, a', b'). The key to our work lies in the observation that in the direct product space $\{\mu\nu\}$, the matrix $\mathcal{A}^{(k)}$ can be diagonalized and expanded into bilinear forms of its eigenvectors [6]. The explicit construction of the (real and positive) eigenvalues $\Lambda^{(s)}(k)$ and of the corresponding right (r) and left (l) eigenvectors $C^{(s\tau)}_{\mu\nu}(k; r)$ and $C^{(s\tau)}_{\mu\nu}(k; l)$ (with τ a label distinguishing degenerate eigenvectors) is lengthy and given in [7]. We only cite the result,

$$\mathcal{A}^{(k)}_{\mu\nu;\rho\sigma} = \frac{1}{N(k)}\sum_{s=0}^{1}\Lambda^{(s)}(k)\sum_{\tau}C^{(s\tau)}_{\mu\nu}(k; l)C^{(s\tau)}_{\rho\sigma}(k; r). \tag{8}$$

Here $N(k) \gg 1$ is a suitably chosen normalization factor related to N_{aa} and $N_{a(a+1)}$.

2. Supersymmetry

Use of the supersymmetry method, although quite different in detail, follows well-established lines [6, 9, 10]. We confine ourselves to mentioning some central points of difference. After averaging the generating function Z for $C(n)$, the exponent of Z carries quartic terms in the integration variables. These terms involve the matrix $\mathcal{A}^{(k)}_{\mu\nu;\rho\sigma}$ of second moments of H_0. The eigenvector expansion equation (8) allows us to use the Hubbard–Stratonovich transformation. We introduce a set of matrices $\sigma^{(k;\tau)}$, eliminate the quartic terms and integrate over the original integration variables. The generating function now involves integrations over the variables in the matrices $\sigma^{(k;\tau)}$. Integration is done using the saddle-point approximation. The saddle points are obtained by putting the hopping terms $v = 0$ and using v as a small parameter afterwards. This step corresponds to the search for low-momentum solutions in the continuum model and is justified in [8].

2.1. Saddle points

The solutions $\sigma^{(k)}_{\rm sp}$ of the resulting saddle-point equations are zero or approximately zero except for those with $k = (a, (a + 1), a, (a + 1))$ and $k = (a, a, a, a)$. The corresponding matrices obey the equations

$$\sigma^{(a(a+1))}_{\rm sp} = \frac{\lambda}{E - \lambda\sigma^{(a(a+1))}_{\rm sp}} \tag{9}$$

and

$$\sigma^{(aa)}_{\rm sp} = N^{-1}_{aa}\,{\rm tr}\left[\left(\frac{\lambda}{E - w - \lambda\sigma^{(aa)}_{\rm sp}}\right)_{\mu\nu}\right]. \tag{10}$$

The saddle-point equation (9) applies when both electrons occupy different slices (a) and ($a + 1$) and, thus, do not interact. The two-body interaction w affects only the saddle-point equation (10) which applies when both electrons occupy the same slice. The diagonal solution for $\sigma^{(a(a+1))}_{\rm sp}$ takes the standard form $\sigma^{(a(a+1))}_{\rm d} = (E/(2\lambda)) \pm {\rm i}\Delta_1(E)$, with $\Delta_1(E) = \sqrt{1 - (E/(2\lambda))^2}$. Here $\Delta_1(E)$ is proportional to the spectral density $\rho_{\rm sp1}(E)$ in box $(a(a + 1))$ and has the shape of a semicircle. The invariance of the saddle-point equation (9)

under pseudounitary graded transformations T implies that the saddle-point manifold has the form [10]

$$\sigma_{\rm sp}^{(a(a+1))} = T^{(a(a+1))}\left[\frac{E}{2\lambda} - {\rm i}\Delta_1(E)L\right](T^{(a(a+1))})^{-1}. \tag{11}$$

Here $L = {\rm diag}(1, 1, -1, -1)$.

The fact that the spectral density $\rho_{\rm sp1}(E)$ in box $(a(a+1))$ has the shape of a semicircle points to a deficiency of the saddle-point approximation: the two electrons in slices a and $(a+1)$ do not interact. The spectrum of each has semicircular shape. The exact spectrum in box $(a(a+1))$ is, thus, the convolution of two semicircles and not a semicircle. This shows that we must take into account loop corrections to the saddle-point approximation. This is done below.

Equation (10) for $\sigma_{\rm sp}^{(aa)}$ contains the two-body interaction w. Diagonalizing w and using a simple geometrical construction shows that equation (10) possesses either N_{aa} real solutions (this applies for $E < E_1$ and for $E > E_2$), or, for $E_1 < E < E_2$, $(N_{aa} - 2)$ real and two complex conjugate solutions $a(E) \pm {\rm i}\Delta_0(E)$. Thus,

$$\sigma_{\rm sp}^{(aa)} = T^{(aa)}[a(E) - {\rm i}\Delta_0(E)L](T^{(aa)})^{-1} \tag{12}$$

where $\Delta_0(E) > 0$ is proportional to the spectral density $\rho_{\rm sp0}(E)$ in box (aa). The essential difference between the saddle-point solutions in equations (11) and (12) lies in the difference between $\Delta_0(E)$ and $\Delta_1(E)$, i.e in the different spectral densities $\rho_{\rm sp0}(E)$ and $\rho_{\rm sp1}(E)$. The interaction w deforms the semicircle $\Delta_1(E)$. Aside from a possible overall shift of the spectrum, w causes $\Delta_0(E)$ to be smaller in the centre of the semicircle, and to extend beyond the endpoints $(-2\lambda, +2\lambda)$ of $\Delta_1(E)$. To illustrate this effect, we calculate $E_{1,2}$ for a weak interaction, using a second-order perturbation expansion in w. This yields

$$E_{1,2} = E_0 \pm (2 + U^2)\lambda. \tag{13}$$

Here $E_0 = (1/N_{aa})\,{\rm tr}(w)$ corresponds to an overall shift of the spectrum while $U^2 = \{(1/N_{aa})\,{\rm tr}(w^2) - [(1/N_{aa})\,{\rm tr}(w)]^2\}/\lambda^2$ causes the spectrum to become wider. Higher orders in the perturbation expansion involve terms ${\rm tr}(w^m)$ with $m > 2$.

We show presently that in our model, it is this deformation of the spectrum in boxes (aa) which changes the localization properties of the two-electron system. Equation (13) then allows us to deduce a qualitative criterion for the onset of interaction-induced effects on localization. To this end, we identify the matrix elements $h_{ij}^{(a)}$ with those of the impurity potential $V_{\rm imp}$. It is easy to see that a qualitative change of the spectral density occurs whenever

$$\langle aa\mu|w^2|aa\mu\rangle_{\rm av} \geqslant \langle ai|(V_{\rm imp})^2|ai\rangle_{\rm av} \tag{14}$$

where the matrix elements are averaged over μ and i, respectively. This is the qualitative criterion announced at the beginning of this paper.

3. Localization

To calculate localization properties, we proceed as in [8]. We expand the effective Lagrangian in the exponent of the generating function Z in powers of v, keeping terms up to second order. The effective Lagrangian takes the form

$$(v/\lambda)^2(3l^2/4)\Delta_0\Delta_1\sum_{j=1}^{K'}{\rm trg}(T^{(j)}L(T^{(j)})^{-1}T^{(j+1)}L(T^{(j+1)})^{-1}). \tag{15}$$

Here the $2K - 1$ boxes (aa) and $(a(a+1))$ of the two-electron problem have been mapped onto $K' = 2K - 1$ slices by putting $j = 2a - 1$ for the boxes (aa) and $j = 2a$ for the boxes

$(a(a+1))$. Expression (15) has exactly the form of the nonlinear sigma model derived for the transport of a single electron through a disordered wire in [8]. Therefore, we identify the coefficient in expression (15) with the dimensionless transport coefficient, i.e., we write

$$T_{\rm sp} = 2\pi\rho_{\rm sp0}(E)v^2\rho_{\rm sp1}(E). \tag{16}$$

We have used the fact that Δ_0 and Δ_1 are proportional to the spectral densities $\rho_{\rm sp0}(E)$ and $\rho_{\rm sp1}(E)$ in boxes (aa) and $(a(a+1))$, respectively. To be precise, we refer to $T_{\rm sp}$ as the saddle-point approximation to the transport coefficient.

The work in [11, 12] has shown that the localization length is proportional to $T_{\rm sp}$. We conclude that the ratio of the localization lengths $\zeta(w \neq 0)$ for a non-vanishing two-body interaction and $\zeta(w = 0)$ for a vanishing two-body interaction is given by

$$\frac{\zeta(w \neq 0)}{\zeta(w = 0)} = \frac{\rho_{\rm sp0}(E)}{\rho_{\rm sp1}(E)}. \tag{17}$$

This result shows how the two-body interaction modifies the localization length. Depending upon the energy considered and on the sizes of the shift and of the widening of the spectrum in box (aa) compared to the unperturbed spectrum in box $(a(a+1))$, the localization length may decrease or increase. In general, we expect an increase near the edge of the unperturbed spectrum, and a decrease in the centre.

It is of interest to compare our result (17) also to the single-particle localization length. In the framework of the model in [8], the latter is given in terms of the dimensionless transport coefficient for single-particle motion. This quantity is formally given by $2\pi\rho_{\rm sp1}v^2\rho_{\rm sp1}$ and, thus, identical to the two-particle transport coefficient as defined by the saddle-point approximation in the absence of any two-body interaction. We emphasize that, in contrast to the two-body problem, the expression for the single-particle problem is exact since all loop corrections vanish in the limit of large l.

3.1. Loop corrections

The result (17) is obtained from the saddle-point approximation, and it was remarked earlier that this approximation is deficient. Therefore, we now study the loop corrections to the saddle-point solution. Writing [10]

$$\sigma^{(k)} = \sigma^{(k)}_{\rm sp} + T^{(k)}\delta\sigma^{(k)}(T^{(k)})^{-1} \tag{18}$$

we expand the σ around the saddle-point solutions $\sigma^{(k)}_{\rm sp}$ and $\overline{Z}$ in powers of $\delta\sigma^{(k)}$, keeping in the exponent only the quadratic terms. As a result, we obtain a set of Gaussian integrals which can be worked out using Wick contraction (see [6]). The loop expansion generates contributions in which, besides the $\delta\sigma^{(k)}$, even powers of v appear to all orders. We consider first terms of zeroth order in v. Inspection shows that, for $n \gg 1$ and to arbitrary order in $\delta\sigma^{(k)}$, the result (17) remains valid after Wick contraction of the $\delta\sigma^{(k)}$. To discuss the terms of order v^2, we recall that the generating function also contains under the integral terms which specify which observable we wish to calculate. In our case, the observable is $C(n)$ in equation (6). We refer to these terms as the source terms. Terms of order v^2 come in two classes. There are corrections of order v^2 which either appear under the same trace(s) as the source terms to begin with, or which become connected to the source terms through Wick contractions of the $\delta\sigma$, and there are other such corrections for which this is not the case. Formally, the latter can be calculated by dropping the source terms altogether. We can show that after Wick contraction, every one of these latter terms takes the form

$$cv^2\sum_{j=1}^{K'}{\rm trg}(T^{(j)}L(T^{(j)})^{-1}T^{(j+1)}L(T^{(j+1)})^{-1}) \tag{19}$$

with c a constant which depends on the particular term considered. Re-exponentiation of this result yields a renormalization of the transport coefficient $T_{\rm sp}$. This is a very satisfactory result. It shows that the localization length becomes renormalized by the loop corrections. As for the size of the renormalization correction, it is probably obvious that we are not able to calculate the values of the coefficients c analytically to every order in the loop expansion. However, we can show that each of these coefficients is determined entirely by two neighbouring boxes, say (aa) and $(a(a+1))$, and is independent of all the other boxes. Any pair of boxes (aa) and $(a(a+1))$ yields for c the same result. Thus, our analytical approach has reduced the calculation of the renormalized transport coefficient to a problem involving only two neighbouring boxes, and not the full Hamiltonian H. This problem is accessible numerically.

To understand the physical origin and meaning of the renormalization terms in this reduced framework, we use the diagrammatic expansion of [13] which is akin to a diagrammatic impurity perturbation expansion. We recall that the saddle-point solution can also be obtained by solving the integral equation for the Green function perturbatively with respect to λ, the strength of the impurity potential. Each term in the perturbation series corresponds to a diagram [13]. Each diagram contains a solid line for the free Green function and a number of wavy lines for the impurity potential. Every wavy line begins and ends on the solid line. The diagram expansion of the saddle-point solution possesses only non-intersecting wavy lines. The loop corrections add diagrams with intersecting wavy lines. The loop corrections also contribute new diagrams containing two solid lines for two Green functions, one for the box (aa) and one for the box $(a(a+1))$ with and without two-body interaction, respectively. The two solid lines are connected by at least one wavy line. In this way, we can see heuristically that the loop correction terms add up to yield the ensemble average of the product of the physical level densities $\rho_0(E)$ and $\rho_1(E)$ in the two boxes. Thus, the sum of the saddle-point value and of all the loop corrections yields for the renormalized transport coefficient T the expression

$$T(w) = 2\pi v^2 \overline{\rho_0(E, w)\rho_1(E)}. \tag{20}$$

The ensemble average in equation (20) can be determined without difficulty by numerical simulation for every given two-body interaction w. We must finally take the continuum limit by letting the longitudinal length d of each slice go to zero and the number K of slices go to infinity. In this limit we have

$$\frac{\zeta(w \neq 0)}{\zeta(w = 0)} = \lim_{d\to 0} \frac{T(w \neq 0)}{T(w = 0)}. \tag{21}$$

The ratio of the localization length ζ for the two-electron system and the localization length $\zeta^{(1)}$ for a single electron, is given by

$$\frac{\zeta(w)}{\zeta^{(1)}} = \lim_{d\to 0} \frac{T(w)}{2\pi v^2(\rho_{\rm sp1}(E))^2}. \tag{22}$$

We believe that in a numerical simulation, the ratio of localization lengths will be fairly independent of d.

4. Summary

We have studied the localization properties of two interacting electrons in a quasi one-dimensional wire. Both electrons move in a disorder potential. Hopping allows the electrons to move along the wire. The two-body interaction has short range. We have reduced the Hilbert space so as to keep the two electrons close to each other.

Our central result is contained in equations (20)–(22). The two-body interaction affects the localization length because it alters the level density in the boxes containing two electrons. Depending on the energy chosen and on the sign of the two-body interaction, the localization length may decrease or increase.

The supersymmetry method does not apply to one-dimensional systems. Hence it is difficult to compare our results with previous numerical work. However, we can directly compare our expression (20) with that in equation (1). The two expressions are clearly different. In our expression (20) we retain the structure typical for the Thouless block-scaling argument with hopping. We predict a change of localization length which depends upon all moments $\mathrm{tr}(w), \mathrm{tr}(w^2), \ldots$ of the two-body interaction. Clarifying the root of the difference clearly requires more work. At this point, we ascribe the difference to the model assumptions used in [1, 2, 4].

References

[1] Shepelyansky D L 1994 *Phys. Rev. Lett.* **73** 2607
See also Dorokhov O N 1990 *Zh. Eksp. Teor. Fiz.* **98** 646 (Engl. transl. 1990 *Sov. Phys.–JETP* **71** 360)
[2] Imry Y 1995 *Europhys. Lett.* **30** 405
[3] von Oppen F and Wettig T 1995 *Europhys. Lett.* **32** 741
von Oppen F, Wettig T and Müller J 1996 *Phys. Rev. Lett.* **76** 491
[4] Frahm K, Müller-Groeling A and Pichard J L 1996 *Phys. Rev. Lett.* **76** 1509
[5] Song P H and von Oppen F 1999 *Phys. Rev.* B **59** 46
[6] Benet L, Rupp T and Weidenmüller H A 2001 *Phys. Rev. Lett.* **87** 010601-1
Benet L, Rupp T and Weidenmüller H A 2001 *Ann. Phys., NY* **292** 67
[7] Richert J and Weidenmüller H A 2001 in preparation
[8] Iida S, Weidenmüller H A and Zuk J 1990 *Ann. Phys., NY* **200** 219
[9] Efetov K B 1983 *Adv. Phys.* **32** 53
[10] Verbaarschot J J M, Zirnbauer M R and Weidenmüller H A 1985 *Phys. Rep.* **129** 367
[11] Zirnbauer M R 1992 *Phys. Rev. Lett.* **69** 1584
[12] Mirlin A D, Müller-Groeling A and Zirnbauer M R 1994 *Ann. Phys., NY* **236** 325
[13] Agassi D, Mantzouranis G and Weidenmüller H A 1975 *Phys. Rep.* **22** 145